Mathematical Computing

Schedule of topics and practical exercises (hands-on for students)

Week 1: Probability

**Introduction and getting started (10 minutes)**

* Discussion: Monty Hall Problem
* Presentation: What is probability?

**Understanding probability (10 minutes)**

* Presentation: Frequentist and Bayesian probability; odds ratios
* Hands-on exercise: Want to make a bet?

**Adding and multiplying probabilities (15 minutes)**

* Presentation: Joint probability; union probability
* Hands-on exercises: Rain and joint probability; rain and union probability

**Conditional probability (20 minutes)**

* Presentation: Conditional probability and colorblindness
* Hands-on exercise: Rain and conditional probability
* Break

**Bayes' theorem (20 minutes)**

* Presentation: Violence and video games
* Hands-on exercise: Medical testing accuracy

**Binomial distribution (10 minutes)**

* Presentation: Binomial distribution
* Hands-on exercise: Airline empty seats

**Normal distribution (15 minutes)**

* Presentation: Normal distribution; quantile functions
* Hands-on exercise: Predict life expectancy of a phone

**Beta distribution (15 minutes)**

* Presentation: Beta distribution and probability of probabilities
* Hands-on exercise: Unfair coin flip

**Other distributions and closing (5 minutes)**

* Presentation: Other distributions (Poisson, exponential)
* Closing and final Q&A

Week 2: Statistics and Hypothesis Testing

**Getting started (5 minutes)**

* Presentation: What to expect; why learn statistics and hypothesis testing?

**The basics (10 minutes)**

* Presentation: Mean and standard deviation; the normal distribution

**Central limit theorem (10 minutes)**

* Presentation: Discovering central limit theorem
* Hands-on exercises: Average die rolls; average samples from uniform distribution

**Population and sample sizes (15 minutes)**

* Presentation: Populations versus samples; standard deviation of the mean
* Hands-on exercises: Sample golden retriever weights; how much is enough?

**P-values and z-tests (20 minutes)**

* Presentation: Discovering the p-value; standard normal distribution and the Z-test
* Hands-on exercise: The tea party problem
* Discussion: What could go wrong using a p-value?
* Break

**Confidence intervals (10 minutes)**

* Presentation: What is a confidence interval?
* Activity: Calculate confidence interval for the mean of unionized salary

**T-distribution (10 minutes)**

* Presentation: T-distribution versus the normal distribution
* Hands-on exercise: Sample size effect on t-distribution

**T-tests (15 minutes)**

* Presentation: Discovering the t-test
* Hands-on exercises: Is my golden retriever underweight? Is my golden retriever overweight?

**Paired and 2-sample t-tests (15 minutes)**

* Presentation: Extending the t-test
* Hands-on exercises: Does this diet work? Did a pricing change increase sales?

**Q&A and closing (10 minutes)**

Week 3: Linear Algebra

**Getting started (5 minutes)**

* Presentation: What is linear algebra? Why learn linear algebra?

**Vectors, combining, and scaling (25 minutes)**

* Presentation: What are vectors? Combining and scaling vectors; span and linear dependence
* Hands-on exercises: Add and scale vectors in NumPy; add and scale vectors

**Transforming vectors and matrices (30 minutes)**

* Presentation: Basis vectors, matrices, the determinant
* Hands-on exercises: Matrices and the determinant in NumPy; transform a vector

**System of linear equations and inverse matrices (15 minutes)**

* Presentation: Solving systems of linear equations with inverse matrices
* Hands-on exercises: Solving systems of linear equations with NumPy; a word problem

**Dot products (20 minutes)**

* Presentation: Understanding dot products, orthogonality
* Hands-on exercises: Dot products with NumPy; execute a dot product

**Matrix decomposition (20 minutes)**

* Presentation: Matrix decomposition, eigenvectors, and eigenvalues
* Hands-on exercises: Matrix decomposition with NumPy; decompose a matrix

**Final questions and closing (5 minutes)**

Week 4: Calculus and Functions

**Number theory (5 minutes)**

* Presentation: Number theory; natural numbers, integers, rational, and irrational numbers
* Hands-on exercise: Identify numeric types

**Mathematical expressions (5 minutes)**

* Presentation: Mathematical expressions; order of operations

**Mathematical functions (10 minutes)**

* Presentation: Intuition behind mathematical functions
* Discussions: Thinking about infinity; linear and nonlinear functions
* Hands-on exercise: How many possible values are there in this function range?

**Exponential functions (10 minutes)**

* Presentation: Rules for exponents; rational and irrational exponents
* Hands-on exercise: Simplify the exponential expressions

**Logarithmic functions (10 minutes)**

* Presentation: Rules for logarithms
* Hands-on exercises: Logarithms in Python; evaluate the logarithmic expressions

**Euler’s number and natural logarithms (25 minutes)**

* Presentation: Euler’s number e; predicting probability of event over time; natural logarithms
* Hands-on exercises: Continuous compounding of interest; evaluate the expressions
* Break

**Derivatives (10 minutes)**

* Presentation: What is a derivative?
* Hands-on exercises: Discover slopes on a function; calculate the slopes on a function

**Partial derivatives (10 minutes)**

* Presentation: Partial derivatives; using tools to calculate partial derivatives

**Gradient descent (10 minutes)**

* Hands-on exercises: Use gradient descent to minimize function; find the minimum

**Calculus integrals (20 minutes)**

* Presentation: Calculating area under curves
* Hands-on exercises: Create integral function from scratch; what’s the area under this function?

**Final Q&A and closing (5 minutes)**

Week 5: Linear Regression

**Getting started (5 minutes)**

* Presentation: What to expect; why learn linear regression?

**Simple linear regression (15 minutes)**

* Presentation: Linear regression; variance and bias
* Hands-on exercises: Fit a simple linear regression using hill climbing; calculate loss

**Multiple linear regression (10 minutes)**

* Presentation: What is multiple linear regression?; fitting techniques beyond hill climbing
* Hands-on exercise: Fit a multivariable linear regression

**Linear regression and scikit-learn (15 minutes)**

* Hands-on exercises: Perform linear regression in scikit-learn; separate training and testing data
* Activity: Perform a linear regression

**Evaluating a regression (25 minutes)**

* Presentation: Mean squared error and mean absolute error; Pearson correlation and correlation matrices; R-squared, coefficient of determination
* Hands-on exercise: Evaluate this linear regression
* Break

**Transforming and feature engineering data (20 minutes)**

* Presentation: Rescaling, standardizing, and normalizing data; feature selection and principal component analysis
* Discussion: Do we always need to transform our data?

**Motivation of regularization (15 minutes)**

* Presentation: The overfitting problem; regularization (L1 and L2)
* Hands-on exercise: Apply regularization to a model

**Stochastic gradient descent (15 minutes)**

* Lecture: Scaling with stochastic gradient descent
* Hands-on exercise: Implement a stochastic gradient descent model

Week 6: Logistic Regression and Classification

**Classification problems (5 minutes)**

* Presentation: Classifying things; different types of classification ML

**Logistic regression intuition (10 minutes)**

* Discussion: Modeling illness after chemical exposure
* Presentation: The sigmoid curve and logistic curve

**Simple logistic regression from scratch (10 minutes)**

* Presentation: Logistic function; maximum likelihood
* Hands-on exercise: Use hill climbing to perform logistic regression

**Multivariable logistic regression (10 minutes)**

* Hands-on exercise: Predict employee retention

**Gradient descent with logistic regression (15 minutes)**

* Presentation: Using gradient descent with logistic regression
* Hands-on exercises: Solve the employee retention problem with gradient descent; choose light or dark font

**Using logistic regression for classification (15 minutes)**

* Presentation: How to use logistic regression for more than one category
* Hands-on exercises: Categorize bank transactions; true/false quick quiz
* Break

**Space Shuttle Challenger (15 minutes)**

* Discussion: The Space Shuttle Challenger disaster; lessons learned
* Hands-on exercise: Model the mishap

**Preparing and rescaling data (5 minutes)**

* Presentation: Normalization, rescaling, standardization, and unit vectors
* Discussion: When do we need to transform data?

**Separating training and testing data (10 minutes)**

* Presentation: K-fold, cross validation, and random fold
* Discussion: Overfitting

**Confusion matrices and ROC/AUC (15 minutes)**

* Hands-on exercise: Identify false positives and false negatives
* Presentation: Why accuracy is a bad measure for classification; confusion matrices; ROC and AUC

**Bayes’ theorem in classification (5 minutes)**

* Discussion: How Bayes’ theorem impacts classification
* Hands-on exercise: Calculate the true positive rate

**Final Q&A and closing (5 minutes)**